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D-branes on some one- and two-parameter Calabi-Yau hypersurfaces

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Abstract: D-branes on one-parameter Calabi-Yau spaces and two-parameter $K3$-fibered Calabi-Yau manifolds are analyzed from both the Gepner model point of view and the geometric perspective. We compute part of the spectrum of the boundary states and comment on the appearance of the D0-brane as well as on non-supersymmetric large volume configurations becoming supersymmetric at the Gepner point.

Keywords: Superstrings and Heterotic Strings, D-branes, Superstring Vacua, Conformal Field Models in String Theory.

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Contents

1. Introduction

D-branes (for review, see eg. [1]) play a pivotal rôle for string dualities. Most work to date has been on D-branes in flat space-time. However, a thorough understanding of string dualities in four dimensions requires D-branes wrapped on cycles of the compactifying manifold. Of those, Calabi-Yau manifolds are of particular interest. The problem of D-branes wrapped on cycles of compact Calabi-Yau three folds was the subject of two very interesting papers [2, 3]. In the first of these papers the simplest Calabi-Yau three-fold, the quintic hypersurface in four-dimensional projective space has been studied in detail. In particular the question whether the geometric picture of D-branes wrapping on cycles survives in the stringy regime was addressed. In the second paper Diaconescu and Römelsberger have extended relevant aspects of this analysis to an elliptically fibred two-parameter Calabi-Yau manifold. The same question has been studied also in [5] and for a non-compact Calabi-Yau manifold in [6]. The general picture has been expounded by Douglas [24]. Even though in these papers important progress has been made, D-branes on Calabi-Yau spaces are far from being understood. The aim of this paper is to apply some aspects of the analysis of [2, 3] to a few more models. These are the three remaining one-parameter models and the two two-parameter K3 fibrations in weighted projective space.

In section 2 we provide some relevant background material about the one-parameter models and compute the intersection matrix on the three-cycles on the mirror as a preparation for the comparison with the computations from superconformal field theory. This will then be done for the $K3$ fibrations in section 3. We then provide in section 4 the basic tool in relating the spectrum of supersymmetric brane configurations at the large volume limit to the periods of the holomorphic three-form of the Calabi-Yau manifold. In particular, we compute the number of moduli of branes wrapped on the $K3$ fiber. In section 5 we discuss the boundary conformal field theory for the Gepner models associated with the Calabi-Yau manifolds under consideration. Finally, in section 6 we put everything together and discuss some interesting boundary states.

Note: the part on the two-parameter $K3$-fibrations has considerable overlap with the recent preprint [32] which appeared while we were in the process of finalizing this paper. Where applicable, our results agree.

2. The 1-parameter models

There are four 1-parameter models of Fermat type: First, there is $\mathbb{P}^4_{1,1,1,1,1}[5]$ known as the quintic in $\mathbb{P}^4$ which has been extensively studied in [10] and [2]. Then there are the degree 6 hypersurfaces $\mathbb{P}^4_{1,1,1,1,2}[6]$ in the weighted projective space $\mathbb{P}^4_{1,1,1,1,2}$, the degree 8 hypersurfaces $\mathbb{P}^4_{1,1,1,1,4}[8]$ in $\mathbb{P}^4_{1,1,1,1,4}$ and the degree 10 hypersurfaces $\mathbb{P}^4_{1,1,1,2,5}[10]$ in $\mathbb{P}^4_{1,1,1,2,5}$ which were explored from the point of view of mirror symmetry in [9] and [7]. Recall that the weighted projective space $\mathbb{P}^4_{w_1,w_2,w_3,w_4,w_5}$ is defined by

$$\mathbb{P}^4_{w_1,w_2,w_3,w_4,w_5} = \frac{\mathbb{C}^5 \setminus \{0\}}{(z_1, z_2, z_3, z_4, z_5) \sim (\lambda^{w_1} z_1, \lambda^{w_2} z_2, \lambda^{w_3} z_3, \lambda^{w_4} z_4, \lambda^{w_5} z_5)} . \tag{2.1}$$

The hypersurfaces X_i are typically given by

$$\begin{array}{lll} X_1 : & z_1^6 + z_2^6 + z_3^6 + z_4^6 + z_5^3 = 0\,, & (z_1 : z_2 : z_3 : z_4 : z_5) \in \mathbb{P}^4_{1,1,1,1,2} \\ X_2 : & z_1^8 + z_2^8 + z_3^8 + z_4^8 + z_5^2 = 0\,, & (z_1 : z_2 : z_3 : z_4 : z_5) \in \mathbb{P}^4_{1,1,1,1,4} \\ X_3 : & z_1^{10} + z_2^{10} + z_3^{10} + z_4^5 + z_5^2 = 0\,, & (z_1 : z_2 : z_3 : z_4 : z_5) \in \mathbb{P}^4_{1,1,1,2,5} \end{array} \tag{2.2}$$

X_1 can be described as triple cover of $\mathbb{P}^3$ branched over a sixtic, while X_2 can be described as a double cover of $\mathbb{P}^3$ branched over an octic [8]. Maybe these descriptions will be useful for studying vector bundles on these Calabi-Yau hypersurfaces by relating them to bundles over $\mathbb{P}^3$, however, so far we have not been able to achieve this.

In order to study the stringy geometry, we consider the mirror manifolds $\widehat{X_i}$ given by the orbifold construction $\{p_i = 0\}/G_i$

$$\begin{array}{ll} p_1 = z_1^6 + z_2^6 + z_3^6 + z_4^6 + z_5^3 - 6\psi z_1 z_2 z_3 z_4 z_5 & G_1 = \mathbb{Z}_3 \times \mathbb{Z}_6^2 \\ p_2 = z_1^8 + z_2^8 + z_3^8 + z_4^8 + z_5^2 - 8\psi z_1 z_2 z_3 z_4 z_5 & G_2 = \mathbb{Z}_2 \times \mathbb{Z}_8^2 \\ p_3 = z_1^{10} + z_2^{10} + z_3^{10} + z_4^5 + z_5^2 - 10\psi z_1 z_2 z_3 z_4 z_5 & G_3 = \mathbb{Z}_{10}^2 \,. \end{array} \tag{2.3}$$

The fundamental objects in our analysis are the periods of the holomorphic 3-form $\widehat{\Omega}$ of $\widehat{X}$. We are interested in relating the large volume limit point in the Kähler moduli space to the Gepner point, hence we have to relate the periods at these two points as in [2]. At the Gepner point there is an enhanced discrete symmetry which acts on the fundamental period $\varpi_0(\psi)$ by $\varpi_j(\psi) = A_G\varpi_0(\psi) = \varpi_0(\alpha^j\psi), j = 0, \ldots, k-1$, where $\alpha^k = 1$. Here $k = 6$, 8, 10 for these three models [7]. Since $b_3(X) = 4$, we see that the periods ϖ_j are not linearly independent. By considering ϖ_j for $|\psi| < 1$ one obtains the following relations between the ϖ_j.

$$\varpi_j + \varpi_{j+2} + \varpi_{j+4} = 0\,, \quad j = 0, 1 \qquad \text{for } \mathbb{P}^4_{1,1,1,1,2}[6] \tag{2.4}$$

$$\varpi_j + \varpi_{j+4} = 0\,, \quad j = 0, 1, 2, 3 \qquad \text{for } \mathbb{P}^4_{1,1,1,1,4}[8] \tag{2.5}$$

$$\begin{aligned} \varpi_j + \varpi_{j+5} &= 0\,, \quad j = 0, 1, \ldots, 4 \\ \varpi_0 + \varpi_2 + \varpi_3 + \varpi_4 + \varpi_5 &= 0\,, \end{aligned} \qquad \text{for } \mathbb{P}^4_{1,1,1,2,5}[10]\,. \tag{2.6}$$

Following [7] we choose as period vectors $\varpi = (\varpi_2, \varpi_1, \varpi_0, \varpi_{k-1})^T$, $k = 6, 8, 10$. On the other hand, the large volume basis will be denoted by $\Pi = (\Pi_6, \Pi_4, \Pi_2, \Pi_0)^T$. Then we have $\Pi = M\varpi$ where $M = KNm$ with K as in [2] and m, N as in [7]. The change of basis for the three models is then, up to an $\mathrm{Sp}(4, \mathbb{Z})$ ambiguity,

$$M = \begin{pmatrix} 0 & -1 & 1 & 0 \\ -1 & 0 & 3 & 2 \\ \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad \text{for } \mathbb{P}^4_{1,1,1,1,2}[6] \tag{2.7}$$

$$M = \begin{pmatrix} 0 & -1 & 1 & 0 \\ -1 & 0 & 3 & 2 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad \text{for } \mathbb{P}^4_{1,1,1,1,4}[8] \tag{2.8}$$

$$M = \begin{pmatrix} 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad \text{for } \mathbb{P}^4_{1,1,1,2,5}[10]\,. \tag{2.9}$$

With the help of these, the intersection form on $H^3(\widehat{X}, \mathbb{Z})$ given in the large volume basis by $\eta_{L14} = -\eta_{L41} = -\eta_{L23} = \eta_{L32} = 1$ can be transformed to the Gepner basis by $\eta_G = M^{-1}\eta_L M^{-1^T}$

$$\eta_G = \begin{pmatrix} 0 & -1 & 2 & 0 \\ 1 & 0 & -1 & 2 \\ -2 & 1 & 0 & -1 \\ 0 & -2 & 1 & 0 \end{pmatrix} \qquad \text{for } \mathbb{P}^4_{1,1,1,1,2}[6]$$

$$\eta_G = \begin{pmatrix} 0 & -1 & 2 & -1 \\ 1 & 0 & -1 & 2 \\ -2 & 1 & 0 & -1 \\ 1 & -2 & 1 & 0 \end{pmatrix} \qquad \text{for } \mathbb{P}^4_{1,1,1,1,4}[8]$$

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$$\eta_G = \begin{pmatrix} 0 & -1 & 1 & 1 \\ 1 & 0 & -1 & 1 \\ -1 & 1 & 0 & -1 \\ -1 & -1 & 1 & 0 \end{pmatrix} \qquad \text{for } \mathbb{P}^4_{1,1,1,2,5}[10]\,. \tag{2.10}$$

Using the relations (2.4), (2.5) and (2.6), we can express η_G as a polynomial I_G in the generator g of the enhanced discrete symmetry groups $\mathbb{Z}_6$, $\mathbb{Z}_8$ and $\mathbb{Z}_{10}$

$$\begin{aligned} I_G &= -g + 2g^2 - 2g^4 + g^5 && \text{for } \mathbb{P}^4_{1,1,1,1,2}[6] \\ I_G &= -g + 2g^2 - g^3 + g^5 - 2g^6 + g^7 && \text{for } \mathbb{P}^4_{1,1,1,1,4}[8] \\ I_G &= -g + g^2 + g^3 - 2g^4 + g^6 - g^7 - g^8 + 2g^9 && \text{for } \mathbb{P}^4_{1,1,1,2,5}[10]\,. \end{aligned} \tag{2.11}$$

We will return these models in section 5.

3. The geometry of the K3 fibrations

We consider the models $\mathbb{P}^4_{1,1,2,2,2}[8]$ which are given as degree 8 hypersurfaces X in $\mathbb{P}^4_{1,1,2,2,2}$ by e.g.

$$z_1^8 + z_2^8 + z_3^4 + z_4^4 + z_5^4 = 0\,, \qquad (z_1 : z_2 : z_3 : z_4 : z_5) \in \mathbb{P}^4_{1,1,2,2,2} \tag{3.1}$$

and the models $\mathbb{P}^4_{1,1,2,2,6}[12]$ which are given as degree 12 hypersurfaces X in $\mathbb{P}^4_{1,1,2,2,6}$ by e.g.

$$z_1^{12} + z_2^{12} + z_3^6 + z_4^6 + z_5^2 = 0\,, \qquad (z_1 : z_2 : z_3 : z_4 : z_5) \in \mathbb{P}^4_{1,1,2,2,6}\,. \tag{3.2}$$

They have been extensively studied in [4] and [12]. Qualitative aspects of these models can and will be discussed together. The distinction between them is only made where necessary. Both have a curve C of A_1 singularities at $z_1 = z_2 = 0$ which are of genus three and two, respectively. Blowing up these singularities gives an exceptional divisor E in X which is a ruled surface. The degree one polynomials generate a linear system $|L|$ which projects X to $\mathbb{P}^1$ with the fibers being $K3$ surfaces. The two divisors L and E generate $H_4(X, \mathbb{Z})$. The degree two polynomials generate another linear system $|H|$ which is related to the first by $|H| = |2L + E|$. The fiber of the ruled surface E will be denoted by l. The intersection of two general members of $|H|$ and $|L|$ will be denoted by h. The two classes h and l generate $H_2(X, \mathbb{Z})$. We choose the generators of the complexified Kähler cone to be (E, L), so that a generic Kähler class is written $K = t_1 E + t_2 L$, where (t_1, t_2) are classical coordinates on the Kähler moduli space of X.

These classes satisfy the following intersection relations. For $\mathbb{P}^4_{1,1,2,2,2}[8]$

$$
\begin{array}{llll}
H^3 = 8 & H^2\cdot L = 4 & H\cdot L^2 = 0 & L^3 = 0, \\
E^3 = -16 & E^2\cdot L = 4 & E\cdot L^2 = 0 & H\cdot E\cdot L = 4 \\
h = \frac{1}{4} H\cdot L & l = \frac{1}{4} H\cdot E & & \\
L\cdot l = 1 & L\cdot h = 0 & H\cdot l = 0 & H\cdot h = 1 \\
E\cdot l = -2, & E\cdot h = 1 & & \\
c_2(X)\cdot E = 8 & c_2(X)\cdot H = 56 & &
\end{array}
\tag{3.3}
$$

and for $\mathbb{P}^4_{1,1,2,2,6}[12]$

$$
\begin{array}{llll}
H^3 = 4 & H^2\cdot L = 2 & H\cdot L^2 = 0 & L^3 = 0, \\
E^3 = -8 & E^2\cdot L = 2 & E\cdot L^2 = 0 & H\cdot E\cdot L = 2 \\
h = \frac{1}{2} H\cdot L & l = \frac{1}{2} H\cdot E & & \\
L\cdot l = 1 & L\cdot h = 0 & H\cdot l = 0 & H\cdot h = 1 \\
E\cdot l = -2, & E\cdot h = 1 & & \\
c_2(X)\cdot E = 4 & c_2(X)\cdot H = 52\,. & &
\end{array}
\tag{3.4}
$$

When we include α' corrections to the classical geometry, we do this by considering the complex structure moduli space of the mirror $\widehat{X}$ which does not get any such corrections. The mirror family $\widehat{X}$ is given by $\{p=0\}/G$ where for $\mathbb{P}^4_{1,1,2,2,2}[8]$ $G = \mathbb{Z}_4^3$ and

$$p = z_1^8 + z_2^8 + z_3^4 + z_4^4 + z_5^4 - 8\psi z_1 z_2 z_3 z_4 z_5 - 2\phi z_1^4 z_2^4\,, \tag{3.5}$$

while for $\mathbb{P}^4_{1,1,2,2,6}[12]$ $G = \mathbb{Z}_6^2 \times \mathbb{Z}_2$ and

$$p = z_1^{12} + z_2^{12} + z_3^6 + z_4^6 + z_5^2 - 12\psi z_1 z_2 z_3 z_4 z_5 - 2\phi z_1^6 z_2^6 \tag{3.6}$$

ψ and ϕ parametrize the moduli space of complex structures of the mirror $\widehat{X}$.

On the Kähler side, the prepotential $\mathcal{F}$ determines the periods $\Pi = (\mathcal{F}^0, \mathcal{F}^1, \mathcal{F}^2, 1, t_1, t_2)^T$ of the holomorphic 3-form $\widehat{\Omega}$ on $\widehat{X}$, where $\mathcal{F}^i = \partial\mathcal{F}/\partial t_i$, $i = 1,2$ and $\mathcal{F}^0 = 2\mathcal{F} - t_i\mathcal{F}^i$. In our two cases we have for $\mathbb{P}^4_{1,1,2,2,2}[8]$ (see [4])

$$\mathcal{F} = -\frac{4}{3}t_1^3 - 2t_1^2 t_2 + \frac{7}{3}t_1 + t_2 + \text{const.} \tag{3.7}$$

and for $\mathbb{P}^4_{1,1,2,2,6}[12]$[1]

$$\mathcal{F} = -\frac{2}{3}t_1^3 - t_1^2 t_2 + \frac{13}{6}t_1 + t_2 + \text{const.} \tag{3.8}$$

[1]see appendix A

Using the intersection numbers (3.3) and (3.4) these can be combined to [11]

$$\mathcal{F} = -\frac{1}{3!}\left(H^3\, t_1^3 + 3H^2\cdot L\, t_1^2 t_2\right) + \frac{t_1}{24}\int_{\widehat{X}} c_2(\widehat{X})H + \frac{t_2}{24}\int_{\widehat{X}} c_2(\widehat{X})L - \frac{\zeta(3)i}{2(2\pi)^3}\chi(\widehat{X})\,. \tag{3.9}$$

As in [3] the choice of (E, L) as the generators of the complexified Kähler cone leads to a non-canonically symplectic intersection form on $H^3(\widehat{X}, \mathbb{Z})$ in the large volume limit

$$\eta_L = \begin{pmatrix} 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 & 0 & 0 \end{pmatrix} \tag{3.10}$$

Hence the period vector in the (E, L) basis becomes $\Pi = (\mathcal{F}^0, \mathcal{F}^1 - 2\mathcal{F}^2, \mathcal{F}^2, 1, t_1, t_2)^T$. Next, we have to relate this period vector to the one at the Gepner point $\varpi = (\varpi_0, \ldots, \varpi_5)^T$ by $\Pi = M\varpi$. Due to the enhanced symmetry, $\mathbb{Z}_k$, where k is the degree of the hypersurface, at this point, the periods $\varpi_j, j = 0, \ldots, k-1$ are not all linearly independent. They satisfy a set of relations which allow us to restrict them to $h^3 = 6$ of these periods. These relations are for $\mathbb{P}^4_{1,1,2,2,2}[8]$

$$\varpi_j + \varpi_{j+2} + \varpi_{j+4} + \varpi_{j+6} = 0 \qquad j = 0, 1 \tag{3.11}$$

and for $\mathbb{P}^4_{1,1,2,2,6}[12]$

$$\varpi_j + \varpi_{j+6} = 0 \qquad j = 0, 1, \ldots, 5 \tag{3.12}$$

The basis transformation matrix M from the Gepner point to the large volume limit point can be obtained by analytic continuation as in [4].[2]

$$M = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & \frac{3}{2} & -2 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{3}{4} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{4} & \frac{1}{4} \end{pmatrix} \qquad \text{for } \mathbb{P}^4_{1,1,2,2,2}[8] \tag{3.13}$$

$$M = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & \frac{3}{2} & -\frac{3}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \qquad \text{for } \mathbb{P}^4_{1,1,2,2,6}[12] \tag{3.14}$$

[2]See also appendix A.

Now we can express the intersection matrix in the basis of the periods at the Gepner point by $\eta_G = M^{-1}\eta_L M^{-1T}$.

$$\eta_G = \begin{pmatrix} 0 & 1 & 0 & -3 & 0 & 3 \\ -1 & 0 & 1 & 0 & -3 & 0 \\ 0 & -1 & 0 & 1 & 0 & -3 \\ 3 & 0 & -1 & 0 & 1 & 0 \\ 0 & 3 & 0 & -1 & 0 & 1 \\ -3 & 0 & 3 & 0 & -1 & 0 \end{pmatrix} \qquad \text{for } \mathbb{P}^4_{1,1,2,2,2}[8] \tag{3.15}$$

$$\eta_G = \begin{pmatrix} 0 & 1 & 0 & -2 & 0 & 1 \\ -1 & 0 & 1 & 0 & -2 & 0 \\ 0 & -1 & 0 & 1 & 0 & -2 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 \\ -1 & 0 & 2 & 0 & -1 & 0 \end{pmatrix} \qquad \text{for } \mathbb{P}^4_{1,1,2,2,6}[12] \tag{3.16}$$

Using the relations (3.11) and (3.12) respectively we can express η_G as a polynomial I_G in the generator g of the enhanced discrete symmetry groups $\mathbb{Z}_8$ and $\mathbb{Z}_{12}$.

$$\begin{aligned} I_G &= g - 3g^3 + 3g^5 - g^7 && \text{for } \mathbb{P}^4_{1,1,2,2,2}[8] \\ I_G &= g - 2g^3 + g^5 - g^7 + 2g^9 - g^{11} && \text{for } \mathbb{P}^4_{1,1,2,2,6}[12]\,. \end{aligned} \tag{3.17}$$

We will return to these expressions in section 5.

4. D-branes and periods

This section follows closely [3] to which we refer for further details. Let $n = (n_6, n_4^1, n_4^2, n_0, n_2^1, n_2^2)$ be an integral vector of $H^3(\widehat{X}, \mathbb{Z})$ which describes the low energy charges of the D-brane. Then the central charge is

$$Z(n) = n \cdot \Pi = n_6\Pi^1 + n_4^1\Pi^2 + n_4^2\Pi^3 + n_0\Pi^4 + n_2^1\Pi^5 + n_2^2\Pi^6\,. \tag{4.1}$$

We want to map these charges to the topological invariants of the corresponding K-theory class ξ which are given by the Chern character $\mathrm{ch}(\xi)$. This is done using the exact form of D-brane Chern-Simons couplings

$$Q = \mathrm{ch}(\xi)\sqrt{\mathrm{td}(X)} \in H^{\mathrm{even}}(X, \mathbb{Z})\,. \tag{4.2}$$

The central charge is

$$Z(K') = \int_X \frac{K'^3 Q^0}{6} - \frac{K'^2 \cdot Q^2}{2} + K' \cdot Q^4 - Q^6\,, \tag{4.3}$$

where $K' = t_1 H + t_2 L$ is the generic Kähler class before the change of basis to (E, L). We apply these general ideas to two cases of D-brane systems. The first are those with nonzero $D6$-brane charge which can be represented by holomorphic vector bundles (more precisely coherent sheaves [23]) V on X. In order for the corresponding D-brane configuration to be supersymmetric V must be stable [30]. From (4.2) we obtain

$$Q = \left(r, \mathrm{c}_1(V), \mathrm{ch}_2(V) + \frac{r}{24}\,\mathrm{c}_2(X), \mathrm{ch}_3(V) + \frac{1}{24}\,\mathrm{c}_1(V)\,\mathrm{c}_2(X) \right) \tag{4.4}$$

and from (4.3)

$$\begin{aligned} Z(Q) = {} & \frac{r}{6}K'^3 - \frac{1}{2}\,\mathrm{ch}_1(V)\cdot K'^2 + \left(\mathrm{ch}_2(V) + \frac{r}{24}\,\mathrm{c}_2(X)\right)K' - \\ & - \left(\mathrm{ch}_3(V) + \frac{1}{24}\,\mathrm{c}_1(V)\,\mathrm{c}_2(X)\right). \end{aligned} \tag{4.5}$$

By comparison one obtains for the Chern classes of V on both models

$$\begin{aligned} r(V) &= n_6 \\ \mathrm{c}_1(V) &= n_4^1 E + n_4^2 L \\ \mathrm{c}_2(V) &= \left(4n_4^1(n_4^2 - n_4^1) - n_2^1\right)h + \left(2(n_4^1)^2 - n_2^2\right) l \\ \mathrm{c}_3(V) &= 2\left(n_4^1\right)^2\left(-4n_4^1 + 3n_4^2\right) + 3n_4^1\left(2n_2^2 - n_2^1\right) - 3n_4^2 n_2^2 - \\ & \quad - 6n_0 - 12n_4^2 + \chi_C n_4^1 , \end{aligned} \tag{4.6}$$

where $\chi_C = -4$ for $\mathbb{P}^4_{1,1,2,2,2}[8]$ and $\chi_C = -2$ for $\mathbb{P}^4_{1,1,2,2,6}[12]$.

The second case consists of systems of D4-branes on the Calabi-Yau manifolds wrapped on holomorphic submanifolds $i : D \hookrightarrow X$ where $D \in H_4(X, \mathbb{Z})$ as has been shown in [15] and [16]. Using the Riemann-Roch-Grothendieck theorem, the central charge associated to the vector Q

$$Q = \left(0, rD, i_*\,\mathrm{c}_1(V) + \frac{r}{2} i_*\,\mathrm{c}_1(D), \mathrm{ch}_2(V) + \frac{1}{2}\,\mathrm{c}_1(V)\,\mathrm{c}_1(D) + \frac{r}{8}\,\mathrm{c}_1(D)^2 + \frac{r}{24}\,\mathrm{c}_2(D)\right) \tag{4.7}$$

becomes [3]

$$\begin{aligned} Z(Q) = {} & -\frac{r}{2}K'^2\cdot D + \left(i_*\,\mathrm{c}_1(V) + \frac{r}{2} i_*\,\mathrm{c}_1(D)\right)K' - \\ & - \mathrm{ch}_2(V) - \frac{1}{2}\,\mathrm{c}_1(V)\,\mathrm{c}_1(D) - \frac{r}{8}\,\mathrm{c}_1(D)^2 - \frac{r}{24}\,\mathrm{c}_2(D) \end{aligned} \tag{4.8}$$

D4-branes wrapped on the exceptional divisor E correspond to BPS states with charge vectors $n = (0, n_4^1, 0, n_0, n_2^1, n_2^2)$ with central charge

$$Z(n) = n_4^1 \mathcal{F}^1 + n_2^1 t_1 + n_2^2 t_2 + n_0 \,. \tag{4.9}$$

Using $c_2(E) = 2\chi_C$ the Chern classes of V can be expressed as

$$\begin{aligned} r(V) &= n_4^1 \\ \mathrm{c}_1(V) &= \left(n_2^1 + \chi_C n_4^1\right)h + \left(n_2^2 - \frac{\chi_C}{2} n_4^1\right) l \\ \mathrm{ch}_2(V) &= -\frac{3}{2}\chi_C n_4^1 - \frac{1}{2} n_2^1 + n_2^2 - n_0 \,. \end{aligned} \tag{4.10}$$

The D4-branes wrapped on the $K3$ fiber L correspond to BPS states with charge vectors $n = (0, 0, n_4^2, n_0, n_2^1, 0)$ with central charge[3]

$$Z(n) = n_4^2 \mathcal{F}^2 + n_2^1 t_1 + n_0\,. \tag{4.11}$$

The Chern classes of V can be expressed as

$$\begin{aligned} r(V) &= n_4^2 \\ \mathrm{c}_1(V) &= n_2^1 h \\ \mathrm{ch}_2(V) &= -2n_4^2 - n_0\,. \end{aligned} \tag{4.12}$$

This gives for the Mukai vector for $K3$ fibers $v(V)$ [23]

$$v(V) = (r(V), c_1(V), r(V) + \mathrm{ch}_2(V)) = \left(n_4^2, n_2^1 h, -n_4^2 - n_0\right) \in H^{\mathrm{even}}(K3, \mathbb{Z})\,. \tag{4.13}$$

There is a natural inner product on the space of Mukai vectors [2]

$$\langle v, v' \rangle = \langle (r, s, \ell), (r', s', \ell') \rangle = s \cdot s' - r \cdot \ell' - \ell \cdot r'\,. \tag{4.14}$$

Applied to our vector $v(V)$ this gives

$$\langle v, v \rangle = 2n_4^2(n_4^2 + n_0) - \frac{1}{\chi_C}\left(n_2^1\right)^2\,, \tag{4.15}$$

where the factor $-1/\chi_C$ arises as follows: $h \cdot h|_L = \frac{1}{\chi_C{}^2} H \cdot H|_L = \frac{1}{\chi_C{}^2} H^2 \cdot L$. A theorem of Mukai [26] shows that the space of coherent simple semistable sheaves with Chern classes specified by Q is smooth and compact and has complex dimension

$$d(n) = \langle v, v \rangle + 2\,. \tag{4.16}$$

This will be used when making the correspondence between supersymmetric D-brane configurations at the large volume limit and the boundary states in the Gepner model to which we now turn.

5. Boundary states in the Gepner model

The Gepner point is characterized by its enhanced discrete (quantum) symmetry and hence by the fact that the corresponding superconformal field theory is exactly solvable. The corresponding boundary conformal field theory has been solved for the rational boundary states in [17] and studied further in [18] and [19]. Here we want to extend the analysis of [2] and [3] to the models described in the previous sections. In particular, we will compute the symplectic intersection form on the BPS charge lattice in the superconformal field theory.

[3] $n_2^2 = 0$ follows e.g. from consistency between (4.8) and (4.11)

The Gepner model $(k_1, k_2, k_3, k_4, k_5)$ is given by the tensor product of $r = 5$ minimal models at level k_j subject to a projection onto states with odd integer U(1) charges and addition of "twisted" sectors in order to keep the theory modular invariant. We include a $k = 0$ factor if present. The superconformal primaries of the minimal models are labelled by 3 integers, (l_j, m_j, s_j) with

$$0 \leq l_j \leq k_j, \quad |m_j - s_j| \leq l_j, \quad s_j \in \{-1, 0, 1, 2\}, \quad l_j + m_j + s_j = 0 \bmod 2\,. \tag{5.1}$$

We also introduce the vectors $\lambda = (l_1, \ldots, l_r)$ for the l_j quantum number, $\mu = (s_0; m_1, \ldots, m_r; s_1, \ldots, s_r)$ for the charges and spin structures. The rational boundary states are constructed by considering each factor separately and then subjecting them to Cardy's consistency condition for modular invariance. They are labelled by $\alpha = (L_j, M_j, S_j)$ and an automorphism Ω of the chiral symmetry algebra giving either A- or B-type boundary conditions. They are

$$|\alpha\rangle\rangle = \frac{1}{\kappa_\alpha^\Omega} \sum_{\lambda,\mu} \delta_\beta \delta_\Omega B_\alpha^{\lambda,\mu} |\lambda, \mu\rangle\rangle_\Omega\,. \tag{5.2}$$

The states $|\alpha\rangle\rangle$ are Cardy states while $|\lambda, \mu\rangle\rangle_\Omega$ are Ishibashi states [25]. κ_α^Ω and $B_\alpha^{\lambda,\mu}$ are given in [17] and [2]. δ_β is a Kronecker delta function enforcing both odd U(1) integral charge and the condition that all factors of the tensor product have the same spin structure. For the B-type boundary states it implies that the physically inequivalent choices for M_j can be described by the quantity

$$M = \sum_{j=1}^{r} \frac{K' M_j}{2k_j + 4}\,, \tag{5.3}$$

CY family	Gepner model	Symmetry group
$\mathbb{P}^4_{1,1,1,1,2}[6]$	$(4, 4, 4, 4, 1)$	$\frac{\mathbb{Z}_6^4 \times \mathbb{Z}_3}{\mathbb{Z}_6} \times \mathbb{Z}_2$
$\mathbb{P}^4_{1,1,1,1,4}[8]$	$(6, 6, 6, 6, 0)$	$\frac{\mathbb{Z}_8^4 \times \mathbb{Z}_2}{\mathbb{Z}_8} \times \mathbb{Z}_2$
$\mathbb{P}^4_{1,1,1,2,5}[10]$	$(8, 8, 8, 3, 0)$	$\frac{\mathbb{Z}_{10}^3 \times \mathbb{Z}_5 \times \mathbb{Z}_2}{\mathbb{Z}_{10}} \times \mathbb{Z}_2$
$\mathbb{P}^4_{1,1,2,2,2}[8]$	$(6, 6, 2, 2, 2)$	$\frac{\mathbb{Z}_8^2 \times \mathbb{Z}_4^3}{\mathbb{Z}_8} \times \mathbb{Z}_2$
$\mathbb{P}^4_{1,1,2,2,6}[12]$	$(10, 10, 4, 4, 0)$	$\frac{\mathbb{Z}_{12}^2 \times \mathbb{Z}_6^2 \times \mathbb{Z}_2}{\mathbb{Z}_{12}} \times \mathbb{Z}_2$

Table 1: The Gepner models under consideration.

where $K' = \mathrm{lcm}\{2k_j + 4\}$. Hence, we will label the B-type boundary states by $|L_1, \ldots, L_r; M; S\rangle\rangle_B$. Due to the symmetry of the superconformal primaries of the minimal models $\chi^{l_j}_{m_j,s_j} = \chi^{k_j - l_j}_{m_j + k_j + 2, s_j + 2}$ one can restrict the values of the L_j to $0 \leq L_j \leq \lfloor k_j/2 \rfloor$. The delta function δ_Ω guarantees that the $|\lambda, \mu\rangle\rangle_\Omega$ appear in the closed string partition function. While not giving any condition for the A-type boundary states, it requires that $m_j = b_j \bmod k_j + 2$ for some b_j. We will denote the set of states which is obtained from a given state $|L_1, \ldots, L_r; M; 0\rangle\rangle_B$ by applying to it the generator of the quantum symmetry as its L-orbit.

The properties of the Gepner models corresponding to the Calabi-Yau spaces in the preceding sections are summarized in table 1

Gepner model	I_A	rk(I_A)
$(4,4,4,4,1)$	$(1-g_2g_3g_4g_5)(1-g_2^5)(1-g_3^5)(1-g_4^5)(1-g_5^2)$	208
$(6,6,6,6,0)$	$(1-g_2g_3g_4g_5)(1-g_2^7)(1-g_3^7)(1-g_4^7)(1-g_5)$	300
$(8,8,8,3,0)$	$(1-g_2g_3g_4g_5)(1-g_2^9)(1-g_3^9)(1-g_4^4)(1-g_5^2)$	292
$(6,6,2,2,2)$	$(1-g_2g_3g_4g_5)(1-g_2^7)(1-g_3^3)(1-g_4^3)(1-g_5^3)$	168
$(10,10,4,4,0)$	$(1-g_2g_3g_4g_5)(1-g_2^{11})(1-g_3^5)(1-g_4^5)(1-g_5)$	254
$(16,16,16,1,0)$	$(1-g_2g_3g_4g_5)(1-g_2^{17})(1-g_3^{17})(1-g_4^2)(1-g_5)$	272

Table 2: Intersection matrices for the A-type boundary states with $L_j = 0$.

In order to find the geometric interpretation of the Gepner model boundary states we consider the intersection form of their charge lattice. In the conformal field theory this can be computed by the Witten index $I_\Omega = \mathrm{tr}_R(-1)^F$ in the open string sector, as was explained in [21] and [22]. This is a topological invariant and hence is unaffected by marginal deformations of the SCFT. In [2] it has been related to the index of the Dirac operator on $V^* \otimes W$, where V and W are the vector bundles on the intersecting branes. For the A-type boundary states it is given by [2]

$$I_A = \frac{1}{C}(-1)^{\frac{S-\tilde{S}}{2}} \sum_{\nu_0=0}^{K-1} \prod_{j=1}^{r} N_{L_j,\widetilde{L}_j}^{2\nu_0+M_j-\widetilde{M}_j} \tag{5.4}$$

and for the B-type boundary states by

$$I_B = \frac{1}{C}(-1)^{\frac{S-\tilde{S}}{2}} \sum_{\{m_a\}} \delta^{(K')}_{\frac{M+\widetilde{M}}{2}+\sum_{j=1}^{r}\frac{K'}{2k_j+4}(m_j+1)} \prod_{j=1}^{r} N_{L_j,\widetilde{L}_j}^{m_j-1}\,, \tag{5.5}$$

where $K = \mathrm{lcm}\{4, 2k_j+4\}$ and $N^l_{L,\widetilde{L}}$ are the extended $\mathrm{SU}(2)_k$ fusion coefficients [2].

Let us first consider the A-type boundary states with $L_j = 0$. Table 2 shows the results for our models where g_i, $i = 2, \ldots, 5$ are the generators of the symmetry group satisfying $g_i^{k_i+2} = 1$. For completeness we have included the elliptic fibration $\mathbb{P}^4_{1,1,1,6,9}[18]$ considered in [3]. It turns out that $\mathrm{rk}(I_A)$ can be related to a geometric quantity

$$\mathrm{rk}(I_A) = \tilde{b}_3(X)\,, \tag{5.6}$$

where $\tilde{b}_3(X)$ denotes the third Betti number of the corresponding Calabi-Yau family without the contributions from non-polynomial deformations of the complex structure [12]. It can be checked that this holds for any Fermat hypersurface X irrespective of $h^{1,1}(X)$. This means that the rank of this intersection matrix counts the number of independent 4-cycles on the mirror Calabi-Yau $\widehat{X}$ except those which are coming from a non-toric blow-up. For the two $K3$ fibrations under consideration the number of non-toric 4-cycles is given by $g = 1 - \chi_C/2$ [20] with the χ_C being introduced in the previous section. It has been recently shown [31] that these complex structure deformations can lead to a superpotential in the non-compact space-time.

Gepner model	I_B	Quantum symmetry
$(4,4,4,4,1)$	$(1-g^5)^4(1-g^4)$	$\mathbb{Z}_6$
$(6,6,6,6,0)$	$(1-g^7)^4(1-g^4)$	$\mathbb{Z}_8$
$(8,8,8,3,0)$	$(1-g^9)^3(1-g^8)(1-g^5)$	$\mathbb{Z}_{10}$
$(6,6,2,2,2)$	$(1-g^7)^2(1-g^6)^3$	$\mathbb{Z}_8$
$(10,10,4,4,0)$	$(1-g^{11})^2(1-g^{10})^2(1-g^6)$	$\mathbb{Z}_{12}$

Table 3: Intersection matrices for the B-type boundary states with $L_j = 0$.

Next we consider the B-type boundary states. Recall that these states are described by the single integer $M \in \mathbb{Z}_{K'}$ and that the g_j for different j are identified. First, we are looking again at the $L_j = 0$ states. In this case, the intersection matrix I_B can be related to the intersection polynomial I_G from the previous sections. It turns out that in all the examples considered, including the quintic in $\mathbb{P}^4$ and the elliptic fibration $\mathbb{P}^4_{1,1,1,6,9}$[18], the relation between the intersection matrix I_B calculated from conformal field theory and the intersection matrix I_G found in (2.11) and (3.17) by using the mirror map at the large volume limit is the same

$$I_B = (1-g)\; I_G\; (1-g^{-1}) \tag{5.7}$$

A convenient way to find the charges for $L_j \neq 0$ has been found in [3]. One replaces each factor $N^{m_j-1}_{L_j,\tilde{L}_j}$ by a factor $n_{L,\tilde{L}}$. Starting from $n_{0,0} = (1-g^{-1})$ one applies to it the linear transformation

$$t_L = {t_L}^T = \sum_{l=-L/2}^{L/2} g^l \tag{5.8}$$

to obtain $n_{L,\tilde{L}} = t_L n_{0,0} {t_{\tilde{L}}}^T$. The charge of the boundary state q_B in the Gepner basis is then given by $q_B t_{L_1} t_{L_2} t_{L_3} t_{L_4} t_{L_5}$. In order to obtain the charges at the large volume limit we substitute the matrix A_L for g, where A_L is given for the different models under discussion in the appendix B.

Before we start looking at the spectrum of B-type boundary states we will mention how to compute the number of boundary marginal operators. For a single boundary state $L = |\,L_j; M; S\rangle\rangle_B$ it is the constant term in [3]

$$P_B = \frac{1}{2}\widetilde{n}_{L_1,L_1}\widetilde{n}_{L_2,L_2}\widetilde{n}_{L_3,L_3}\widetilde{n}_{L_4,L_4}\widetilde{n}_{L_5,L_5} - \rho\,, \tag{5.9}$$

where $\widetilde{n}_{L_j,L_j} = |n_{L_j,L_j}|$ and $\rho = 2^{\gamma-1}$. γ counts the number of L_j which are equal to $k_j/2$. This is due to the symmetry $|L_j\rangle\rangle_B = |k_j - L_j\rangle\rangle_B$ in a L_j-orbit which halves the number of states except for $L_j = k_j/2$. In the expression for ρ, $k_j = 0$ factors are not taken into account.

6. Boundary states and D-branes

In this section we put everything together and establish an explicit correspondence between the D-branes described by rational boundary states in the Gepner model and supersymmetric D-brane configurations on the Calabi-Yau spaces discussed above.

For the 1-parameter models considered here we are faced with the same problem as for the quintic: there is a general lack of knowledge of vector bundles on these spaces. Hence we can only consider some particular aspects. In [2] it was noticed that there was no D0 brane in the spectrum of the quintic at the Gepner point. Even though this only means that there is no corresponding rational boundary state for the D0-brane, it was argued later on in [24] that there might be a line of marginal stability which might prevent its existence at the Gepner point at all. However, this seems not to be generically the case, as has been shown in [3] and as we will show here. Let us discuss some interesting states in our models. In $\mathbb{P}^4_{1,1,1,1,2}[6]$ we do not find the D0-brane either, however there is an interesting state in the orbit of $L = |1,1,0,0,0\rangle\rangle_B$ which is a D2-brane with charge $Q_2 = 3$. This state has 8 marginal operators in the Gepner model.

For $\mathbb{P}^4_{1,1,1,1,4}[8]$ some interesting boundary states are listed in table 4 (there are many more boundary states which, however, we do not discuss here) Here we find the D0-brane in the last line. Since the number of vacua is two, we might think of the boundary state as two different D0-branes, each of which would have 3 moduli. This matches with the expectation that the moduli space of the D0-brane is the whole Calabi-Yau manifold. As was argued in [3] this might be a sign of a Coulomb branch in the worldvolume theory in which the gauge group is $U(1)^2$. In the first line we also find the D2-brane wrapping some 2-cycle while there was no pure D4-brane found. Particularly interesting are the states in the second and third line. They describe D6-$\overline{\text{D0}}$ and D4-D2 bound states respectively. A D4-D2 system is known to break supersymmetry completely in flat space. The D4-D2 potential which is difficult to determine on a curved space is expected to approach the flat space result in the large volume limit. Hence this configuration should become non-supersymmetric and repulsive for a sufficiently big radius which prevents in particular the formation of a bound state. This gives a further example of a supersymmetric boundary state at the Gepner point decaying into a non-supersymmetric combination of D-branes at large volume point. This phenomenon was first observed in [3].

L-orbit	charge	#moduli	#vacua
$\|1,1,0,0,0\rangle\rangle_B$	$(0,0,2,0)$	7	1
$\|2,0,0,0,0\rangle\rangle_B$	$(1,0,0,-2)$	6	1
$\|2,1,0,0,0\rangle\rangle_B$	$(0,1,2,0)$	11	1
$\|3,0,0,0,0\rangle\rangle_B$	$(0,0,0,2)$	6	2

Table 4: Some boundary states of $\mathbb{P}^4_{1,1,1,1,4}[8]$.

Next, we consider some important boundary states for $\mathbb{P}^4_{1,1,1,2,5}[10]$ which are listed in table 5. Again we find the D0-brane in the spectrum of the boundary states. In this model it appears in two different ways: Once as a single D0-brane in the first line with the expected dimension of the moduli space and once as two different D0-branes in the last line, precisely in the same way as in the case above. Furthermore we find in addition to the D2-brane wrapping some 2-cycle in the second line also a D4-brane wrapping some 4-cycle. Finally there is a supersymmetric bound state of a D4-D2 system at the Gepner point corresponding to a non-supersymmetric configuration at the large volume point.

L-orbit	charge	#moduli	#vacua
$\vert 0,0,0,1,0\rangle\rangle_B$	$(0,0,0,1)$	3	1
$\vert 2,0,0,0,0\rangle\rangle_B$	$(0,0,2,0)$	4	1
$\vert 2,0,0,1,0\rangle\rangle_B$	$(0,1,0,0)$	8	1
$\vert 2,0,0,1,0\rangle\rangle_B$	$(0,1,1,0)$	8	1
$\vert 4,0,0,0,0\rangle\rangle_B$	$(0,0,0,2)$	6	2

Table 5: Some boundary states of $\mathbb{P}^4_{1,1,1,2,5}[10]$.

For the remainder of this section we turn to the more interesting $K3$-fibrations. Table 6 gives all the boundary states which describe brane configurations wrapped on the $K3$ fiber of $\mathbb{P}^4_{1,1,2,2,6}[12]$. The criterion is that their charge vector is of the general form $(0,0,n_4^2,n_0,n_2^1,0)$. The anti-branes whose charge vector has the opposite overall sign are not given in table 6.

The BPS condition for the charges is [23] $d \geq 0$ with $r > 0$ or $r(V) = 0$, $\mathrm{c}_1(V) > 0$ or $r = \mathrm{c}_1 = 0$, $\mathrm{ch}_2(V) < 0$. This is satisfied for all the charges above. Note that all the states in a given orbit lead to the same dimension d which provides a check on (4.15).

There are some further boundary states of particular interest listed in table 7. The first one of these boundary states corresponds to a D4-brane wrapped around the exceptional divisor which is a ruled surface E.

$$r(V) = 0\,, \qquad \mathrm{c}_1 = 2E\,, \qquad \mathrm{c}_2(V) = 4l - 8h\,, \qquad \mathrm{c}_3(V) = -36 \tag{6.1}$$

L-orbit	Mukai vector $v = (n_4^2, n_2^1, -n_4^2 - n_0)$	#moduli	#vacua	d
$\vert 1,0,0,0,0\rangle\rangle_B$	$(2,-2,1)$ $(1,0,1)$ $(1,-2,2)$	1	1	0
$\vert 3,0,0,0,0\rangle\rangle_B$	$(1,0,-1)$ $(0,2,-1)$ $(1,-2,0)$	5	1	4
$\vert 3,0,1,0,0\rangle\rangle_B$	$(1,-4,1)$ $(2,-2,-1)$ $(1,2,-2)$	9	1	8
$\vert 3,0,1,1,0\rangle\rangle_B$	$(0,6,-3)$ $(3,-6,0)$ $(3,0,-3)$	21	1	20
$\vert 5,0,0,0,0\rangle\rangle_B$	$(2,0,0)$ $(0,0,2)$ $(2,-4,2)$	6	2	2
$\vert 5,0,1,0,0\rangle\rangle_B$	$(2,-4,0)$ $(2,0,-2)$ $(0,4,-2)$	14	2	10
$\vert 5,0,1,1,0\rangle\rangle_B$	$(2,-8,2)$ $(4,-4,-2)$ $(2,4,-4)$	30	2	26
$\vert 5,0,2,0,0\rangle\rangle_B$	$(0,4,0)$ $(4,-4,0)$ $(0,4,-4)$	20	4	10
$\vert 5,0,2,1,0\rangle\rangle_B$	$(4,-8,0)$ $(4,0,-4)$ $(0,8,-4)$	44	4	34
$\vert 5,0,2,2,0\rangle\rangle_B$	$(4,-12,4)$ $(4,-4,-4)$ $(4,4,-4)$	64	8	42

Table 6: All boundary states describing brane configurations wrapped on the $K3$ fiber of $\mathbb{P}^4_{1,1,2,2,6}[12]$.

There are only a few results known about semistable sheaves on ruled surfaces. Their application to our problem is work in progress. The second one corresponds to a D2-brane wrapped on the 2-cycle h which lies at the intersection of the divisors H and L.

L-orbit	charges	#moduli	#vacua
$\vert 0,0,2,2,0\rangle\rangle_B$	$(0,2,0,0,0,0)$	8	4
$\vert 2,2,0,0,0\rangle\rangle_B$	$(0,0,0,0,4,0)$	11	1
$\vert 4,0,0,0,0\rangle\rangle_B$	$(1,0,0,2,0,0)$	6	1
$\vert 5,0,0,0,0\rangle\rangle_B$	$(0,0,0,2,0,0)$	6	2
$\vert 5,0,0,0,0\rangle\rangle_B$	$(2,0,0,2,0,0)$	6	2

Table 7: Some further boundary states of the model $\mathbb{P}^4_{1,1,2,2,6}[12]$.

$$r(V) = 0\,, \qquad c_1(V) = 0\,, \qquad c_2(V) = -4h\,, \qquad c_3(V) = 0 \tag{6.2}$$

The fourth one is the D0-brane which has already appeared as a stable sheaf on the $K3$ fiber.

$$r(V) = 0\,, \qquad c_1(V) = 0\,, \qquad c_2(V) = 0\,, \qquad c_3(V) = -12 \tag{6.3}$$

As in the case of some of the one-parameter models it describes two different D0-branes, each of them having three moduli. Next we are going to consider bound states of D0- and D6-branes. They are described by the boundary states in the third and and fifth row of the table. The first of these is completely analogous to the one observed in [3] for the elliptic fibration.

$$r(V) = 1\,, \qquad c_1(V) = 0\,, \qquad c_2(V) = 0\,, \qquad c_3(V) = -12 \tag{6.4}$$

The supersymmetric boundary state in the Gepner model decays into a non-supersymmetric configuration of D-branes at the large volume limit. The authors of [3] have given an interesting interpretation from the point of view of the mirror $\widehat{X}$ which we shortly repeat here. In [27] it has been proposed that mirror symmetry is T-duality when X and $\widehat{X}$ admit special lagrangian fibrations with T-dual T^3, $\widehat{T}^3$ fibers. Mirror symmetry now maps the D6-brane on X to D3-branes wrapping the base B of the fibration while the D0-branes are mapped to D3-branes wrapping the $\widehat{T}^3$ fiber. The above decay process now tells us that the corresponding homology class $B + \widehat{T}^3$ should not support a special lagrangian cycle in a neighborhood of the large complex structure limit. It should support it instead in a region of the moduli space of $\widehat{X}$ which is mapped to a neighborhood of the Gepner point of X by mirror symmetry. These are the phase transitions of special lagrangian cycles under the deformation of the complex structure of $\widehat{X}$ which have been studied in [28] and [29]. In our example we find another possibility when we consider the boundary state in the last row.

$$r(V) = 2\,, \qquad c_1(V) = 0\,, \qquad c_2(V) = 0\,, \qquad c_3(V) = -12 \tag{6.5}$$

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L-orbit	Mukai vector $v=(n_4^2, n_2^1, -n_4^2-n_0)$	#moduli	#vacua	d
$\vert 1,0,0,0,0\rangle\rangle_B$	$(3,-4,1)$ $(-3,8,-3)$ $(1,-4,3)$ $(-1,0,-1)$	1	1	0
$\vert 3,0,0,0,0\rangle\rangle_B$	$(0,4,-2)$ $(2,-4,0)$	7	1	6
$\vert 3,0,1,0,0\rangle\rangle_B$	$(2,-8,2)$ $(2,0,-2)$	14	2	10
$\vert 3,0,1,1,0\rangle\rangle_B$	$(4,-8,0)$ $(0,8,-4)$	28	4	18
$\vert 3,0,1,1,1\rangle\rangle_B$	$(4,0,-4)$ $(4,-16,4)$	56	8	34

Table 8: All boundary states describing brane configurations wrapped on the $K3$ fiber of $\mathbb{P}^4_{1,1,2,2,2}[8]$.

It is also a bound state of a D0- and a D6-brane, but now there are two vacua which can be interpreted as two bound states of one D6- and one D0-brane with three moduli each.

For $\mathbb{P}^4_{1,1,2,2,2}[8]$ the boundary states of table 8 correspond to a D-brane wrapped on the $K3$ fiber. Again, all these boundary states satisfy the BPS condition. There are no single D0-branes, no D4-branes wrapped on the exceptional divisor E, nor are there bound states of D0- and D6-branes.

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A. Analysis of the periods of the degree 12 hypersurface in $\mathbb{P}^4_{1,1,2,2,6}$

Here we perform the calculation of the monodromy matrices for the model $\mathbb{P}^4_{1,1,2,2,6}[12]$ according to [4]. The fundamental period is given by

$$\begin{aligned}\varpi_0(\psi,\phi) &= \sum_{r,s=0}^{\infty} \frac{(12r+6s)!(-2\phi)^s}{(6r+3s)!((2r+s)!)^2(r!)^2 s!(12\psi)^{12r+6s}} \\ &= \sum_{n=0}^{\infty} \frac{(6n)!(-1)^n}{(n!)^3(3n)!(12\psi)^{6n}} u_n(\phi), \qquad \left|\frac{\phi\pm 1}{864\psi^6}\right| < 1\,, \end{aligned} \tag{A.1}$$

where

$$u_n(\phi) = (2\phi)^n \sum_{r=0}^{[n/2]} \frac{n!}{(r!)^2(n-2r)!(2\phi)^{2r}}\,. \tag{A.2}$$

After extension of the definition of $u_n(\phi)$ to complex values ν for n we write the period as an integral of Barne's type in order to obtain an expression which is valid for small ψ.

$$\varpi_0(\psi,\phi) = \frac{1}{2\pi i}\int_C \mathrm{d}\nu \frac{\Gamma(6\nu+1)\Gamma(-\nu)}{\Gamma(3\nu+1)\Gamma^2(\nu+1)}(12\psi)^{-6\nu}u_\nu(\phi),$$
$$-\pi < \arg\left(\frac{864\psi^6}{\phi\pm 1}\right) < \pi\,. \tag{A.3}$$

For $|\phi\pm 1| < |864\psi^6|$ we can close the contour to the right and obtain (A.1) as a sum of the residues of the poles of $\Gamma(-\nu)$. For $|\phi \pm 1| > |864\psi^6|$ the contour can be closed to the left giving a sum over the residues of the poles of $\Gamma(6\nu+1)$. Defining

$$\varpi_j(\psi,\phi) = \mathcal{A}^j\varpi_0(\psi,\phi) = \varpi_0(\alpha^j\psi,(-1)^j\phi)\,, \tag{A.4}$$

with $\alpha^{12}=1$ we find

$$\varpi_j(\psi,\phi) = -\frac{1}{6}\sum_{m=1}^{\infty}\frac{(-1)^m\alpha^{jm}\Gamma(m/6)}{\Gamma(m)\Gamma(1-m/2)\Gamma^2(1-m/6)}(12\psi)^m u_{-m/6}((-1)^j\phi)\,. \tag{A.5}$$

The factors $\Gamma(1-m/2)$ and $\Gamma(1-m/6)$ in the denominator determine the relations between the periods as follows

$$\varpi_j + \varpi_{j+6} = 0\,. \tag{A.6}$$

Hence we have for $\mathcal{A}:\varpi \longrightarrow A_G\varpi$

$$A_G = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{A.7}$$

To compute the monodromy of these periods around $\phi = 1$ we have to continue the ϖ_j to large values of ψ. Following [4] we write the variable of summation as $m = 6n + r$

$$\begin{aligned}\varpi_{2j} &= -\frac{1}{6\pi^3}\sum_{r=1}^{5}(-1)^r\alpha^{2jr}\sin\left(\frac{\pi r}{2}\right)\sin^2\left(\frac{\pi r}{6}\right)\xi_r\\ \varpi_{2j+1} &= -\frac{1}{6\pi^3}\sum_{r=1}^{5}(-1)^r\alpha^{(2j+1)r}\sin\left(\frac{\pi r}{2}\right)\sin^2\left(\frac{\pi r}{6}\right)\eta_r\,,\end{aligned} \tag{A.8}$$

where

$$\begin{aligned}\xi_r(\psi,\phi) &= \frac{1}{2i}\int_C \mathrm{d}\nu\frac{\Gamma^3(-\nu)\Gamma(-3\nu)}{\Gamma(-6\nu)}(12\psi)^{-6\nu}\frac{u_\nu(\phi)}{\sin\left(\pi(\nu+\frac{r}{6})\right)}\\ \eta_r(\psi,\phi) &= -\frac{1}{2i}\int_C \mathrm{d}\nu\frac{\Gamma^3(-\nu)\Gamma(-3\nu)}{\Gamma(-6\nu)}(12\psi)^{-6\nu}\frac{u_\nu(\phi)\sin\left(\pi(\nu+\frac{r}{6})\right)+u_\nu(-\phi)\sin(\frac{\pi r}{6})}{\sin\left(\pi(\nu+\frac{r}{6})\right)\sin(\pi\nu)}\end{aligned} \tag{A.9}$$

An analogous computation to that in [4] gives for $\mathcal{B} : \varpi \longrightarrow B_G \varpi$

$$B_G = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 \\ -1 & 1 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 2 & -2 \\ 0 & 0 & -1 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} . \tag{A.10}$$

To compute the monodromy around the conifold point we repeat again the steps of [4] and obtain for $\mathcal{T} : \varpi \longrightarrow T_G \varpi$

$$T_G = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 \\ -2 & 2 & 0 & 1 & 0 & 0 \\ 2 & -2 & 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{pmatrix} . \tag{A.11}$$

In [4] the large complex structure limit point in the moduli space was determined to be at the intersection of the two divisors $\mathrm{D}_{(1,0)} \rightsquigarrow (A_G T_G B_G)^{-1}$ and $\mathrm{D}_{(0,-1)} \rightsquigarrow (A_G T_G)^{-2}$. If we take (H, L) as the basis for the Kähler cone, i.e. $B + iJ = t_1 H + t_2 L$, then the coordinates t_i can be related to the periods ϖ_j by

$$t_1 = -\frac{1}{2} + \frac{\varpi_2 + \varpi_4}{2\varpi_0} \tag{A.12}$$

$$t_2 = \frac{1}{2} + \frac{\varpi_1 - \varpi_2 + \varpi_3 - \varpi_4 + \varpi_5}{2\varpi_0} \tag{A.13}$$

With the ansatz for the prepotential $\mathcal{F}$

$$\mathcal{F} = -\frac{1}{6}\left(4{t_1}^3 + 6{t_1}^2 t_2\right) + \frac{1}{2}\left(\alpha {t_1}^2 + 2\beta t_1 t_2 + \gamma {t_2}^2\right) + \left(\delta - \frac{2}{3}\right) t_1 + \varepsilon t_2 + \mathrm{const} \tag{A.14}$$

the constants δ and ϵ can be fixed the same way as in [4][4] while the constants α, β and γ can be chosen appropriately by an $\mathrm{Sp}(6, \mathbb{Z})$ transformation.

$$\alpha = 0\,, \qquad \beta = 0\,, \qquad \gamma = 0\,, \qquad \delta = \frac{17}{6}\,, \qquad \varepsilon = 1\,. \tag{A.15}$$

This choice differs from the one in [4] and is dictated by, as has been argued in [2], the fact that the state which becomes massless at the mirror of the conifold point should correspond to the "pure" six-brane with large volume charges $Q = (1, 0, 0, 0, 0, 0)$,

[4] In [4] δ and ε appear to have the wrong sign

following [13] and [14]. Finally we give the expression for the matrix M

$$M = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ \frac{3}{2} & \frac{3}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \tag{A.16}$$

B. The monodromy matrices around the Gepner point

$$A_L = \begin{pmatrix} -3 & -1 & -6 & 4 \\ -3 & 1 & 3 & 3 \\ 1 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{pmatrix} \qquad \text{for } \mathbb{P}^4_{1,1,1,1,2}[6]$$

$$A_L = \begin{pmatrix} -3 & -1 & -4 & 4 \\ -2 & 1 & 2 & 2 \\ 1 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{pmatrix} \qquad \text{for } \mathbb{P}^4_{1,1,1,1,4}[8]$$

$$A_L = \begin{pmatrix} -2 & -1 & -1 & 3 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{pmatrix} \qquad \text{for } \mathbb{P}^4_{1,1,1,2,5}[10]$$

$$A_L = \begin{pmatrix} -1 & 0 & 1 & -2 & 0 & 0 \\ 4 & -1 & 0 & 4 & -4 & -4 \\ -2 & 1 & 1 & -2 & 4 & 2 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & -1 \end{pmatrix} \qquad \text{for } \mathbb{P}^4_{1,1,2,2,2}[8]$$

$$A_L = \begin{pmatrix} -1 & 0 & 1 & -2 & 0 & 0 \\ 2 & -1 & 0 & 2 & -2 & -2 \\ -1 & 1 & 1 & -1 & 2 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & -1 \end{pmatrix} \qquad \text{for } \mathbb{P}^4_{1,1,2,2,6}[12]$$

These can be obtained from the corresponding matrices in [7] and [4] by applying the linear transformation M in (2.7), (2.8), (2.9), (3.13) and (3.14) respectively to them.

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